# Physics of magma segregation processes

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Abstract—Partial melting occurs in the mantle due to pressure reduction or by volumetric heating. As a result magma is produced along grain boundaries at depths up to 100 km or more. Differential buoyancy drives this magma upwards by a porous flow mechanism. The small magma conduits on grain boundaries may coalesce to form rivulets of magma in the asthenosphere. The reduction in density due to the presence of magma may induce diapiric flows in the mantle. When the ascending magma reaches the base of the lithosphere it is likely to form pools of magma. Buoyancy driven magma fractures appear to be to the only mechanism by which this magma can penetrate through the lithosphere. How these fractures are initiated remains a subject of speculation.

## **INTRODUCTION**

PLATE TECTONICS provides a framework for understanding magmatic activity on and in the Earth. However, many important aspects of magmatic processes are poorly understood. A large fraction of the Earth's volcanism is associated with midocean ridges. Over a large fraction of the ridge system this is essentially a *passive process*. As the plates diverge mantle rock must ascend by solid state creep to fill the gap. Average upper mantle rock is sampled by this random, unsteady process. As the mantle rock ascends melting occurs due to the adiabatic decompression. A substantial fraction of the resulting magma ascends to near surface magma chambers due to the differential buoyancy. This magma solidifies to form the oceanic crust.

Volcanism is also associated with subduction zones. The origin of this volcanism is still a subject of controversy. Friction on the slip zone between the descending lithosphere and overlying plate provides some heat. However, as the rock solidus is approached, the viscosity drops to such low levels that direct melting is not expected (YUEN et al., 1978). Offsets in the linear volcanic chains associated with subduction zones correspond with variations in the dip of the Benioff zones. This is strong evidence that the magma is produced on or near the boundary between the descending lithosphere and overlying plate. This magma then ascends vertically 150  $\pm$  25 km to form the distinct volcanic edifices observed on the surface. How the magma collects, ascends and feeds the distinct centers is poorly understood.

Volcanism also occurs within plate interiors. A fraction of this volcanism may be associated with ascending plumes within the mantle. These plumes may be the result of instabilities of the thermal boundary layer at the base of the convecting upper mantle, either at a depth of 670 km for layered

mantle convection or above the core-mantle boundary for whole mantle convection (ALLEGRE and TURCOTTE, 1985). Pressure release melting would produce the observed volcanism. However, in many cases, the magma must penetrate the full thickness of the cold lithosphere in order to produce surface volcanism.

We will consider three mechanisms for magma migration: (1) porous flow, (2) diapirism and (3) fracture. Aspects of the magma migration problem have been reviewed by SPERA (1980) and TUR-COTTE (1982).

## POROUS FLOW

Adiabatic decompression will lead to partial melting of the mantle on grain boundaries. Experimental studies (WAFF and BULAU, 1979, 1982; COOPER and KOHLSTEDT, 1984, 1986; FUJII *et al.*, 1986) and theoretical calculations (VON BARGEN and WAFF, 1986) have shown that the melt forms an interconnected network of channels along triple junctions between grains. Magma will migrate upwards along these channels due to differential buoyancy. This configuration suggests that a porous flow model may be applicable for magma migration in the region where partial melting is occurring.

Porous flow models for magma migration have been given by FRANK (1968), SLEEP (1974), TUR-COTTE and AHERN (1978a,b), and AHERN and TUR-COTTE (1979). Implicit in these models is the assumption that the matrix is free to collapse as the magma migrates upwards. The validity of this assumption has been questioned by MCKENZIE (1984). It has been shown by SCOTT and STEVENSON (1984) and by RICHTER and MCKENZIE (1984) that the coupled migration and compaction equations have unsteady solutions (solitons). However, it was shown approximately by AHERN and TURCOTTE (1979) and rigorously by RIBE (1985) that compaction is not important in mantle melting by decompression. The melting of the ascending mantle takes place sufficiently slowly that the crystalline matrix is free to collapse by solid state creep processes. It is appropriate to assume that the fluid pressure is equal to the lithostatic pressure. Any difference between the hydrostatic (fluid) pressure and the lithostatic pressure will drive the buoyant fluid upwards. The conclusion is that the unsteady solutions are not relevant to magma migration in the mantle.

In order to quantify magma migration in a porous solid matrix, Darcy's law gives (TURCOTTE and SCHUBERT, 1982, p. 414):

$$\Delta v = \frac{b^2 \phi \Delta \rho g}{24\pi \eta_{\rm m}} \tag{1}$$

where  $\Delta v$  is the magma velocity relative to the solid matrix, b is the grain size,  $\phi$  is the volume fraction of magma (porosity),  $\Delta \rho$  is the density difference between the magma and the solid matrix, g is the acceleration of gravity, and  $\eta_m$  is the magma viscosity. Taking b = 2 mm,  $\Delta \rho = 600 \text{ kg/m}^3$  and  $g = 10 \text{ m/s}^2$ , the magma migration velocity is given as a function of the magma viscosity in Figure 1 for several values of porosity.



The upward flux of magma can be related to the upward flux of the component to be melted by

$$\frac{v_{\rm m}\phi}{3} = v_{\rm s}f\tag{2}$$

where  $v_{\rm m}$  is the magma velocity,  $v_{\rm s}$  is the upward velocity of the mantle prior to melting, and *f* is the fraction of the mantle that is melted. By using Equation (1) to relate  $v_{\rm m}$  and  $\phi$  and taking f = 0.25, the appropriate values of  $v_{\rm m}$  and  $\phi$  are given in Figure 1 for specified values of  $\eta_{\rm m}$  and  $v_{\rm s}$ . For example, with  $v_{\rm s} = 1$  cm/year and  $\eta_{\rm m} = 10$  Pa s, we find that  $\phi = 0.003$  and  $v_{\rm m} = 10^{-7}$  m/s (300 cm/yr) as indicated by the solid dot in Figure 1.

Caution should be used in the application of the porous flow model. The theory assumes a uniform distribution of small channels and in fact the small channels may coalesce to form larger channels as the magma ascends in much the same way that streams coalesce to form rivers. There is some evidence that this occurs when ice melts.

#### DIAPIRISM

Diapirism has long been considered as a mechanism for magma migration (MARSH, 1978, 1982; MARSH and KANTHA, 1978). Two types of diapirism must be considered. First, consider magma that has been segregated from the mantle by the porous flow mechanism. The ascending magma is likely to pool at the base of the lithosphere. The question is whether the buoyancy can drive the pooled magma through the lithosphere as a diapir. In order for this to occur, the country rock must be displaced by solid state creep so that the viscosity of the medium through which the diapir is passing governs the velocity of ascent. A second question is whether the residual partial melt fraction in the asthenosphere can induce diapirism.

In order to consider these problems we study the idealized problem of a low viscosity, low density fluid sphere ascending through a viscous medium due to buoyancy. The velocity of ascent for this Stokes problem is given by (TURCOTTE and SCHU-BERT, 1982, p. 267)

$$v_{\rm d} = \frac{r^2 g \Delta \rho \phi}{3\eta_{\rm s}} \tag{3}$$

where, r, is the radius of the sphere and,  $\eta_s$ , is the viscosity of the medium. The ascent velocity is given as a function of the viscosity in Figure 2 for several values of the sphere radius and melt fraction and  $\Delta \rho = 600 \text{ kg/m}^3$ .

Because the lithosphere certainly has a viscosity of at least  $10^{24}$  Pa s, the migration velocity spheres





FIG. 2. The ascent velocity of a low viscosity, spherical diapir,  $v_d$ , as a function of the viscosity of the medium through which it is rising,  $\eta_s$ , for several values of the sphere radius, r, and porosity  $\phi$ . This result from Equation (3) assumes  $\Delta \rho = 600 \text{ kg/m}^3$ .

with a radius less than 100 km is extremely small. The diapir will solidify before it can migrate through a cold lithosphere. If the path through the lithosphere was heated so that the viscosity was lowered, then significant migration may occur. But what is the mechanism for heating if magma cannot penetrate the lithosphere? The conclusion is that diapirism is not a viable mechanism for the migration of magma through the lithosphere.

One possibility for this type of migration is that heat transported from the magma or heating by viscous dissipation, or both, softens the medium through which the diapir is passing. Studies of this problem (RIBE, 1983; EMERMAN and TURCOTTE, 1984; OCKENDON *et al.*, 1985) show that these mechanisms do not significantly help in the penetration of cold lithosphere.

Results given in Figure 2 can also be used to determine whether the melt fraction in the asthenosphere can induce diapirism. Asthenosphere viscosities may be as low as  $10^{18}$  Pa s although  $10^{21}$ Pa s is probably a better estimate. With  $\eta_s = 10^{21}$ Pa s,  $\phi = 0.01$ , and r = 100 km, we find  $v_d = 1$  cm/ yr. Under these conditions, diapirism due to partial melt is relatively unimportant. However, with  $\eta_s$ =  $10^{18}$  Pa s, the velocity may approach  $10^3$  cm/yr. Thus it is not possible to make a definitive conclusion on the relative role of diapirism within the asthenosphere. To treat this problem satisfactorily, the mantle convection problem must be solved simultaneously with the melting and porous flow migration problems and this has not been done so far.

## MAGMA FRACTURE

A third mechanism for magma migration is fluid fracture. This is probably the only mechanism capable of transporting magma through the cold lithosphere. The common occurrence of dikes is direct observational evidence for the existence of magma fractures. The dynamics of pressure-driven fluid fractures has been studied by SPENCE and TURCOTTE (1985) and EMERMAN *et al.* (1986). These authors showed that dike propagation is restricted by the viscosity of the magma and by the fracture resistance of the media, but in most geological applications, the fracture resistance can be neglected.

The vertical transport of magma over large distances (10–100 km) is almost certainly driven by the differential buoyancy of the magma. One of the most spectacular examples of buoyancy driven magma fracture is a kimberlite eruption; the required velocities are estimated to be 0.5-5 m/s (PASTERIS, 1984). Buoyancy–driven fluid fractures as a mechanism for magma migration have been studied by WEERTMAN (1971), ANDERSON and GREW (1977), ANDERSON (1979) and SECOR and POLLARD (1975). The dynamics of a buoyancy driven fluid fracture have been given by SPENCE *et al.* (1986). These authors found that the stress intensity factor (fracture resistance) plays an essential role in the solution.

The propagation of a buoyancy-driven fluid fracture is governed by the equations for the fluid flow in the crack; the equation governing the deformation of the elastic medium, and the equation relating the empirically derived stress intensity factor to the tip curvature of the crack. Mathematical details for the two-dimensional problem have been given by SPENCE *et al.* (1986). These authors find a steady-state solution for a propagating crack. A universal shape is obtained in terms of non-dimensional variables. The non-dimensional crack width 2H is given as a function of the non-dimensional distance from the crack tip in Figure 3. The non-dimensional variables are related to actual variables by

$$H = \frac{h}{h_{\infty}}, \quad X = \left(\frac{[1-\nu]\Delta\rho g}{h_{\infty}\mu}\right)^{1/2} x \tag{4}$$

where  $2h_{\infty}$  is the width of the tail,  $\mu$  the shear modulus,  $\nu$  Poisson's ratio,  $\Delta\rho$  the density difference and g the acceleration of gravity. One of the important results of the analysis is the requirement that the crack must have an infinitely long constant width tail. The flow in the tail is a balance between the buoyancy driving force and the viscous resisting



FIG. 3. Shape of an upward propagating magma fracture. The non-dimensional half-width H is given as a function of the non-dimensional distance from the crack tip X. The non-dimensional variables are defined in Equation (4).

force. Thus the velocity of propagation of the crack, c, is related to the half-width of the tail  $h_{\infty}$  by

$$c = \frac{gh_{\infty}^2 \Delta \rho}{3\eta_{\rm m}} \tag{5}$$

for laminar flow in the crack and

$$c = \frac{7.71 h_{\infty}^{5/7} (\Delta \rho g)^{4/7}}{\rho_{\rm m}^{3/7} \eta_{\rm m}^{1/7}} \tag{6}$$

for turbulent flow.

For the tail of the crack, the non-dimensional half-width is  $H_{\infty} = 1$ . The maximum crack half-width is H = 1.8975 and this occurs at X = 1.152. A steady-state solution to this problem is found only for a single value of the non-dimensional stress intensity factor,  $\lambda$ , defined by

$$\lambda = \frac{1}{2^{1/2}} \left( \frac{(1-\nu)}{\mu h_{\infty}} \right)^{3/4} \frac{K_{\rm c}}{(g\Delta\rho)^{1/4}} = 1.3078 \tag{7}$$

where  $K_c$  is the critical stress intensity factor. Solving for the half-width of the tail gives

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$$h_{\infty} = 0.440 \frac{K_{\rm c}^{4/3}(1-\nu)}{\mu(g\Delta\rho)^{1/3}}.$$
 (8)

When the critical stress intensity factor is specified, the half-width of the crack is obtained from Equation (8). The propagation velocity is then obtained from Equation (5) for laminar flow and Equation (6) for turbulent flow.

The critical stress intensity factor,  $K_c$ , is a measure of the fracture toughness of the material. Cracks can propagate at low velocities ( $\sim 10^{-4}$  m/s) by the mechanism of stress corrosion; in this range  $K < K_c$ . However, when  $K = K_c$ , crack propagation becomes catastrophic and the propagation velocity may rapidly accelerate to a large fraction of the speed of sound. The values of  $K_c$  for a variety of rocks have been obtained in the laboratory. This work has been reviewed by ATKINSON (1984). Measured values for granite range from  $K_c = 0.6$  to 2.2 MN/m<sup>3/2</sup> and from  $K_c = 0.8$  to 3.3 MN/m<sup>3/2</sup> for basalts. A measured value for dunite is  $K_c = 3.7 \text{ MN/m}^{3/2}$ . These values were obtained at atmospheric pressure. The influence of pressure on the stress intensity factor is difficult to predict. SCHMIDT and HUDDLE (1977) found a factor of four increase at a pressure of 62 MPa for Indiana limestone. Thus, it is difficult to specify values of the critical stress intensity factor for regions of the crust and mantle where buoyancydriven magma fracture is occurring.

By taking  $\mu = 2 \times 10^{10}$  Pa,  $\nu = 0.25$ ,  $\Delta \rho = 300$  kg/m<sup>3</sup> and g = 10 m/s<sup>2</sup> the half-width of the tail from Equation (8) is given as a function of the critical stress intensity factor in Figure 4. Propagation velocities from Equations (5) or (6) are given as a function of the critical stress intensity factor in Figure 5. Results are given for magma viscosities  $\eta_m = 0.1$ , 3, 100 Pa s. We use the laminar result equation (5) for Reynolds numbers Re =  $\rho_m ch_{\infty}/\eta_m$  less than 10<sup>3</sup> and the turbulent result Equation (6) for Reynolds numbers greater than 10<sup>3</sup>. The corre-



FIG. 4. The half width of the crack tail  $h_{\infty}$  as a function of the critical stress intensity factor  $K_c$  from Equation (8) by assuming  $\mu = 2 \times 10^{10}$  Pa,  $\nu = 0.25$ ,  $\Delta \rho = 300$  kg/m<sup>3</sup>.



FIG. 5. Crack propagation velocity, c, as a function of the critical stress intensity factor,  $K_c$ , for several values of the magma viscosity,  $\eta_m$ . Laminar flow from Equation (5) and turbulent flow from Equation (6).

sponding values of the volumetric flow rate  $V = 2ch_{\infty}$  per unit crack length are given in Figure 6.

CARMICHAEL *et al.* (1977) have shown that the entrainment of xenoliths in basaltic flows implies velocities, c, of at least c = 0.5 m/s. A variety of studies indicate that basaltic magma migrates upward at velocities in the range c = 0.5 to 5 m/s. A typical viscosity for a basaltic magma is  $\eta_m = 0.1$  Pa s. From Figures 4 and 5 the corresponding range of the stress intensity factor is  $K_c = 20$  to 100 MN/m<sup>3/2</sup> and the range of tail half-widths is  $h_{\infty} = 5$  to 50 mm. From Figure 6 the corresponding range of flow rates per unit length is V = 0.0025 to  $0.25 \text{ m}^2/\text{s}$ .

Studies of the Kilauea Iki eruption on an island of Hawaii during 1959–1960 give flow rates of 50 to 150 m<sup>3</sup>/s (WILLIAMS and MCBIRNEY, 1979, pp. 232–233). Taking a flow rate of 100 m<sup>3</sup>/s, the range of flow rates per unit length given above, V = 0.0025to 0.25 m<sup>2</sup>/s, correspond to crack lengths between 400 m and 4 km. These appear to be reasonable lengths for the crack feeding Kilauea Iki. Although the above comparison appears reasonable, there is conflicting evidence whether the flow of magma at the surface represents the flow through the lithosphere. The role of shallow magma chambers in storing magma is poorly understood.

As a specific example of a magma fracture that transports magma through the lithosphere we take:  $\mu = 2 \times 10^{10}$  Pa,  $\nu = 0.25$ ,  $\Delta \rho = 600$  kg/m<sup>3</sup>, g = 10m/s<sup>2</sup>,  $\eta_{\rm m} = 0.1$  Pa s, and  $K_{\rm c} = 50$  MN/m<sup>3/2</sup>. We find that  $h_{\infty} = 0.0167$  m and c = 2.86 m/s. To convert the shape given in Figure 3 into actual distances, we require h = 0.0167H m and x = 272X m. If the length of the crack is 1 km the volume flux through the lithosphere is  $95.5 \text{ m}^3/\text{s}$ .

## CONCLUSIONS

Pressure release melting produces magma on grain boundaries. Magma on triple junctions between grains produce an interconnected network of channels. Under mantle conditions the magma drains rapidly upwards due to differential buoyancy and the residual solid matrix collapses due to solid state creep. Magma that is produced over a depth range of approximately 50 km mixes during the vertical ascent to produce the magma reaching the base of the lithosphere.

At mid-ocean ridges there is no lithosphere and the ascending magma can form directly the oceanic crust. However, off the axis at ocean trenches and at intraplate volcanic centers, the magma must penetrate through the lithosphere. Magma fracture is a mechanism for the rapid transport of magma through the lithosphere. A theory for steady state magma fracture through the lithosphere is given. A typical fracture width is 2 cm and fracture velocity is 5 m/s. Under these conditions relatively little magma is lost by solidification during ascent. The existence of dikes is evidence that magma fracture is a pervasive mechanism. The extensive systems of dikes in deeply eroded terranes is evidence that dikes are the dominant mechanism for the ascent of magma through the crust. Kimberlite eruptions are direct evidences that magma fracture can transport magma through the lithosphere.

How magma fractures initiate remains a matter of speculation. It may be possible to form magma fractures as magmas perculating in small channels



FIG. 6 Volumetric flow rate per unit crack length, V, as a function of the critical stress intensity factor,  $K_c$ , for several values of the magma viscosity,  $\eta_m$ .

coalesce to form larger channels as suggested by FOWLER (1985). Alternatively, magma may pool at the base of the lithosphere. When the pool reaches a critical size the buoyancy forces may initiate a magma fracture that subsequently drains the pool.

Acknowledgements—The author would like to acknowledge the many contributions of D. A. Spence to our understanding of the magma fracture problem. This research was supported by the Division of Earth Sciences, National Science Foundation under grant EAR-8518019. This is contribution 830 of the Department of Geological Sciences, Cornell University.

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